#### Seiji Tomita and Oliver Couto

#### Abstract

In this paper, we proved that there are infinitely many parametric solutions of  $n(x^4 + y^4) = z^4 + w^4$ .

# 1. Introduction

In 2016 Izadi and Nabardi[2] showed  $x^4 + y^4 = 2(u^4 + v^4)$  has infinitely many integer solutions.

In 2016 Izadi and Nabardi[2] showed  $x^4 + y^4 = 2(u^4 + v^4)$  has infinitely many integer solutions. They used a specific congruent number elliptic curve namely  $y^2 = x^3 - 36x^2$ . In 2019 Janfada and Nabardi[3] showed that a necessary condition for *n* to have an integral solution for the equation  $x^4 + y^4 = 0$   $(u^4 + v^4)$  and gave a parametric solution. They gave the numeric solutions for  $n = 41, 136, 313, 1028, 1201, 3281, \cdots$ .

In 2020 Ajai Choudhry[1], Iliya Bluskov and Alexander James showed  $(x_1^4 + x_2^4)(y_1^4 + y_2^4) = z_1^4 + z_2^4$  has infinitely many parametric solutions. They gave the numeric solutions for  $n = 17,257,626,641,706,1921,\cdots$ . Inspired by Choudhry's article, we showed if  $n = (a^4 + b^4)/2$  then  $n(x^4 + y^4) = z^4 + w^4$  has infinitely many parametric solutions.

We consider n be an integer. Using four different methods we showed  $n(x^4 + y^4) = z^4 + w^4$  has infinitely many integer solutions. First method is similar as Choudhry's one, second method is using another identity. First method is similar as Choudhry's one, second method is using another identity. Moreover, we showed the third method through an example  $17(x^4 + y^4) = z^4 + w^4$ . In addition, we showed the fourth method through an example  $97(x^4 + y^4) = z^4 + w^4$ . Finally, we gave the numeric solutions for  $n = 2, 8, 17, 41, 82, 97, 113, 136, 137, 146, 178, 193, \cdots$  with n < 1000.

#### 2. First method

**Theorem 1**  $(x^4 + y^4)(a^2 + b^2) = z^4 + w^4$  has infinitely many parametric solutions, where  $(a,b) = (\frac{m^2 - n^2}{2}, \frac{m^2 + n^2}{2}), m, n$  are arbitrary integer and have same parity.

Proof.

We use Brahmagupta's identity

$$(A^{2} + B^{2})(C^{2} + D^{2}) = (AC + BD)^{2} + (AD - BC)^{2}$$

We obtain

$$(x^4 + y^4)(a^2 + b^2) = (x^2a + y^2b)^2 + (x^2b - y^2a)^2$$

Hence we have to find the rational solution of  $x^2a + y^2b = u^2$  and  $x^2b - y^2a = v^2$ .

Parametric solution of first equation is  $(x, y) = (k^2 - 2mk + 4a - 3m^2, -k^2 - 2mk + 4a - m^2)$  where  $a + b = m^2$ . Substitute it to second equation, we obtain

Let  $a = \frac{m^2 - n^2}{2}$  and  $U = \frac{1}{k}$ , then we obtain

$$V^{2} = (5m^{4}n^{2} + 4n^{6})U^{4} + (4m^{5} + 8mn^{4})U^{3} - 2m^{2}n^{2}U^{2} - 4m^{3}U + n^{2}$$
(1)

Since equation (1) has a point Q(U, V) = (0, n), then equation (1) is birationally equivalent to an elliptic curve E.

$$E: Y^{2} - 4m^{3}YX/n + (8nm^{5} + 16mn^{5})Y = X^{3} - 2m^{2}(n^{4} + 2m^{4})X^{2}/(n^{2}) + (-20m^{4}n^{4} - 16n^{8})X + 104m^{6}n^{6} + 32m^{2}n^{10} + 80m^{10}n^{2}$$

Transformation is given,

$$\begin{split} U &= \frac{2n^2 X - 4m^2 n^4 - 8m^6}{Yn} \\ V &= \frac{n^4 X^3 - 6n^6 X^2 m^2 - 12 X^2 m^6 n^2 + 28n^8 m^4 X + 32n^4 m^8 X + 32m^{12} X - 16m^9 Yn + 16n^{9} mY + 16n^{12} X - 104n^{10} m^6 - 32n^{14} m^2 - Y^2 n^3 \\ X &= \frac{2n V + 2n^2 - 4m^3 U}{U^2} \\ Y &= \frac{4n^3 V + 4n^4 - 8n^2 m^3 U - 4n^4 m^2 U^2 - 8m^6 U^2}{U^3 n} \end{split}$$

The point corresponding to point Q is  $P(X,Y) = \left(\frac{2m^2(n^4+2m^4)}{n^2}, \frac{-16m(-m^8+n^8)}{n^3}\right)$ . According to the Nagell-Luts theorem, since the point P is a point of infinite order then we can obtain infinitely many rational points on E. Thus we can obtain infinitely many rational solutions of (1). Therefore  $(x^4 + y^4)(a^2 + b^2) = z^4 + w^4$  has infinitely many parametric solutions. The proof is completed.

When m, n have opposite parity, problem links to  $h(x^4 + y^4) = 2(z^4 + w^4)$  where  $a^2 + b^2 = h/2$ .

Example  $(a,b) = \left(\frac{m^2 - n^2}{2}, \frac{m^2 + n^2}{2}\right)$  m, n are arbitrary integer and have same parity.  $2Q(U) = \frac{-2n^2m}{n^4 + m^4}$ 

$$\begin{split} x &= -n^4 + 4m^2n^2 + m^4 \\ y &= n^4 + 4m^2n^2 - m^4 \\ z &= (3n^4 + m^4)m \\ w &= (n^4 + 3m^4)n \end{split}$$

$$3Q(U) = \frac{2mn^2(n^4 + 3m^4)}{n^8 - 8m^4n^4 - m^8}$$

$$\begin{aligned} x &= m^{12} + 12n^2m^{10} - 19m^8n^4 + 40n^6m^6 + 19m^4n^8 + 12n^{10}m^2 - n^{12} \\ y &= -m^{12} + 12n^2m^{10} + 19m^8n^4 + 40n^6m^6 - 19m^4n^8 + 12n^{10}m^2 + n^{12} \\ z &= m(m^{12} + 41m^8n^4 + 27m^4n^8 - 5n^{12}) \\ w &= n(-n^{12} - 41m^4n^8 - 27m^8n^4 + 5m^{12}) \end{aligned}$$

Numerical example: (m, n) < 10

Table 1: Solutions of $(x^4 + y^4)(a^2 + b^2) = z^4 + w^4$						
m	n	$a^2 + b^2$	х	У	Z	W
3	1	41	29	11	63	61
4	2	136	31	1	76	98
5	1	313	181	131	785	469
5	3	353	361	89	1085	1467
6	4	776	209	79	774	1036
7	1	1201	649	551	4207	1801
7	3	1241	1021	139	4627	5463
7	5	1513	1669	781	7483	9785
8	2	2056	319	191	2072	1538
8	6	2696	751	401	3992	5094
9	1	3281	1721	1559	14769	4921
9	5	3593	3509	541	18981	25385
9	7	4481	5009	2929	30969	38647

### 3. Second method

**Theorem 2**  $(x^4 + y^4)\frac{m^4+1}{2} = z^4 + w^4$  has infinitely many parametric solutions, where *m* is arbitrary odd number.

Proof.

We use an identity

$$p(t+1)^4 + p(t)^4 = (t^2 + at + b)^2 + (ct^2 + dt + e)^2$$

where

$$(a,b,c,e,p) = (d+2,\frac{1}{2d}+1,1+d,\frac{1}{2d},1+d+\frac{1}{2d^2})$$

So, we look for the integer solutions  $z^2 = t^2 + (d+2)t + \frac{1}{2d} + 1$  and  $w^2 = (1+d)t^2 + dt + \frac{1}{2d}$ . By parameterizing the first equation and substituting the result to second equation, then we obtain quartic equation.

Let  $d = m^2 - 1$  and U = 1/k then

$$V^{2} = (4m^{6} + 5m^{2})U^{4} + (-8m^{4} - 4)U^{3} - 2m^{2}U^{2} + 4U + m^{2}$$
<sup>(2)</sup>

This quartic equation is birationally equivalent to an elliptic curve below.

$$E: Y^2 - 4YX/m + (8m + 16m^5)Y = X^3 - 2(m^4 + 2)X^2/(m^2) + (-20m^4 - 16m^8)X + 104m^6 + 32m^{10} + 80m^2 + 104m^6 + 32m^2 + 104m^2 + 104m^2$$

Transformation is given,

$$\begin{split} U &= \frac{2m^2 X - 4m^4 - 8}{Ym} \\ V &= \frac{m^4 X^3 - 6m^6 X^2 - 12X^2 m^2 + 28m^8 X + 32m^4 X + 32X + 16Ym - 16m^9 Y + 16m^{12} X - 32m^{14} - 104m^{10} - 80m^6}{Y^2 m^3} \\ X &= \frac{2mV + 2m^2 + 4U}{U^2} \\ Y &= \frac{4m^3 V + 4m^4 + 8m^2 U - 4m^4 U^2 - 8U^2}{U^3 m} \end{split}$$

It has a point  $P(X,Y) = (\frac{2(m^4+2)}{m^2}, \frac{-16(-1+m^8)}{m^3})$ . According to the Nagell-Luts theorem, since the point P is a point of infinite order then we can obtain infinitely many rational points on E.

Thus we can obtain infinitely many rational solutions of (2) using group law. Therefore  $(x^4 + y^4)\frac{m^4+1}{2} = z^4 + w^4$  has infinitely many parametric solutions. The proof is completed.

When m is even number, problem links to  $(m^4 + 1)(x^4 + y^4) = 2(z^4 + w^4)$ .

Example  $n = (m^4 + 1)/2$ m is odd number.

$$x = m4 + 4m2 - 1$$
$$y = m4 - 4m2 - 1$$
$$z = 3m4 + 1$$
$$w = (m4 + 3)m$$

$$\begin{split} x &= m^{12} + 12m^{10} - 19m^8 + 40m^6 + 19m^4 + 12m^2 - 1\\ y &= m^{12} - 12m^{10} - 19m^8 - 40m^6 + 19m^4 - 12m^2 - 1\\ z &= 5m^{12} - 27m^8 - 41m^4 - 1\\ w &= m(m^{12} + 41m^8 + 27m^4 - 5) \end{split}$$

4. Example for 
$$41(x^4 + y^4) = z^4 + w^4$$

We show a numerical example of first method. Let (a,b,m) = (4,5,3) then  $(x^4 + y^4)(a^2 + b^2) = z^4 + w^4$  is reduced to

$$V^2 = U^4 - 108U^3 - 18U^2 + 996U + 409$$

Quartic equation is birationally equivalent to an elliptic curve E.

$$E: Y^2 + XY + Y = X^3 - X^2 - 27X + 26$$

Transformation is given,

$$U = \frac{4Y + 29X - 10}{X - 46}$$

$$V = \frac{16X^3 - 2208X^2 - 3392X + 13744 - 9840Y}{(X - 46)^2}$$

$$X = \frac{V + U^2 - 54U + 5}{32}$$

$$Y = \frac{UV + U^3 - 83U^2 + 99U - 29V + 175}{128}$$

The rank of elliptic curve E is 1 and has generator P(X, Y) = (6, -11) using SAGE. We can find infinitely many rational points on the curve E using the group law. Thus the multiples nP, n = 2, 3, ... give infinitely many points as follows.

Table 2: Solutions of  $41(x^4 + y^4) = z^4 + w^4$ 

nP	Х	У	Z	W	
2P	29	11	63	61	
3P	17909	5149	37623	38699	
4P	229422601	214213319	663306603	282177719	
5P	81840455152441	-86237007592439	252933880274523	61172008172039	

5. Example for 
$$17(x^4 + y^4) = z^4 + w^4$$

The previous methods can't give the rational solution of  $17(x^4 + y^4) = z^4 + w^4$ . Hence we show the third method of giving the rational solution to the equation  $(x^4 + y^4)(a^2 + b^2) = z^4 + w^4$ . We consider the simultaneous equation as follows.

$$\begin{cases} z^2 - w^2 - 5x^2 - 3y^2 = 2t(4x^2 + w^2 - y^2)\\ (w^2 + z^2 + 5x^2 + 3y^2)t = -(x^2 + w^2 + 4y^2) \end{cases}$$

Eliminating  $z^2$ , we get

$$(1+10t+8t^2)x^2 + (4+6t-2t^2)y^2 + (2t^2+1+2t)w^2 = 0$$
(3)

Parametric solution of (3) is  $(x, y, w) = (2(k-2)(k-3), (k-1)(3k-7), -(25+5k^2-22k)).$ 

$$z^{2} = \frac{-w^{2} + 17y^{2}}{4}$$
$$= \frac{4(2k^{2} - 5k + 1)(4k^{2} - 15k + 13)}{(-5 + k^{2})^{2}}$$

Hence cosider the quartic equation (4)

$$V^2 = 8U^4 - 50U^3 + 105U^2 - 80U + 13 \tag{4}$$

Quartic equation (4) has a rational point Q(U, V) = (2, 1), then this quartic equation is birationally equivalent to an elliptic curve below.

 $E: Y^2 = X^3 - 91X + 330$  with rank is 1 and generator=(7, -6). Transformation is given,

$$U = \frac{6X - 36 + 2Y}{Y + 2X - 12}$$

$$V = \frac{-18X^2 - 114 + X^3 + 91X}{(Y + 2X - 12)^2}$$

$$X = \frac{2V + 6 - U^2}{U^2 - 4U + 4}$$

$$Y = \frac{12V - 108 + 180U - 90U^2 - 4VU + 14U^3}{U^3 - 6U^2 + 12U - 8}$$

The rank of elliptic curve E is 1 and has generator P(X, Y) = (7, -6) using SAGE. We can find infinitely many rational points on the curve E using the group law. Thus the multiples nP, n = 2, 3, ...give infinitely many points as follows.

Table 3: Solutions of  $17(x^4 + y^4) = z^4 + w^4$ 

nP	х	У	Z	W
2P	3120	1921	2242	6529
3P	18418554	88538885	176117272	95896333
4P	87733253643360	108376421998081	198203611434238	206237591201281
5P	12509563104278834954874	6446124521923428875525	3117838409641509334568	25836199364300466735373

6. Example for 
$$97(x^4 + y^4) = z^4 + w^4$$

The previous methods can't give the rational solution of  $97(x^4 + y^4) = z^4 + w^4$ . Hence we show the fourth method of giving the rational solution to the equation  $(x^4 + y^4)(a^2 + b^2) = z^4 + w^4$ . According to Richmond's theorem[4], existence of solution for diophantine equation  $ax^4 + by^4 + cz^4 + dw^4 = 0$  are known if *abcd* is square number. He proved other solution is derived from a known solution. Repeting this process, we can obtain infinitely many integer solutions. We use a known solution (x, y, z, w) = (112, 71, 10, 37) obtained by brute force as follows. Let x = pt + 112, y = qt + 71, z = rt + 10, w = t + 37.

We obtain

$$\begin{array}{l} (q^4 + p^4 - 97r^4 - 97)t^4 + (-3880r^3 + 284q^3 - 14356 + 448p^3)t^3 \\ + (-796758 - 58200r^2 + 30246q^2 + 75264p^2)t^2 + (-19653364 + 5619712p - 388000r + 1431644q)t = 0 \end{array}$$

In order to set the coefficient of t and  $t^2$  to zero, we obtain

$$q = \frac{-135744p}{42103} + \frac{491693}{42103}$$
$$r = \frac{-22422}{2965} + \frac{7672p}{2965}$$

Hence we obtain

$$t = \frac{-1238771307200}{68835707869p - 174887242544}$$

Take p=1 then  $(q, r) = (\frac{355949}{42103}, \frac{-2950}{593}).$ 

Finally, we obtain (x, y, z, w) = (174887242544, 240033770927, 68026751110, 68835707869).

Thus we can obtain other integer solution by using this new solution as a known solution. According to Richmond's theorem, all solutions of Table 4 have infinitely many integer solutions.

 $\begin{array}{l} \text{Smallest: } Min \ n(x^4+y^4) \\ \text{Brute force search range: } n < 1000, (x,y,z,w) < 50000 \\ n \equiv 1,2,8,9 \ \ \text{mod 16 is to be searched.} \end{array}$ 

$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	able 4:	Solution	ns of $n($	$x^4 + y^4$ )	$=(z^4+u)$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	n	х	У	Z	W
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	2	7	20	21	19
411131 $82$ $219$ $320$ $1011$ $247$ $97$ $10$ $37$ $112$ $71$ $113$ $1$ $2$ $6$ $5$ $136$ $1$ $1$ $4$ $2$ $137$ $29$ $5$ $99$ $31$ $146$ $1$ $2$ $7$ $3$ $178$ $1$ $2$ $7$ $5$ $193$ $18$ $43$ $159$ $80$ $226$ $1$ $8$ $31$ $7$ $241$ $1$ $2$ $8$ $1$ $257$ $4$ $15$ $52$ $49$ $313$ $1$ $1$ $5$ $1$ $328$ $7$ $3$ $30$ $8$ $337$ $15$ $34$ $147$ $26$ $353$ $1$ $1$ $5$ $3$ $386$ $1$ $2$ $9$ $1$ $401$ $1$ $2$ $9$ $1$ $401$ $1$ $2$ $9$ $4$ $433$ $8$ $13$ $53$ $50$ $466$ $2$ $43$ $181$ $151$ $482$ $5$ $6$ $31$ $7$ $577$ $175$ $188$ $929$ $848$ $578$ $24$ $37$ $187$ $85$ $593$ $1$ $2$ $10$ $3$ $6673$ $161$ $26$ $820$ $139$ $706$ $17$ $28$ $149$ $13$ $712$ $15$ $13$ $86$ $3$	8	19	21	40	14
82 $219$ $320$ $1011$ $247$ $97$ $10$ $37$ $112$ $71$ $113$ $1$ $2$ $6$ $5$ $136$ $1$ $1$ $4$ $2$ $137$ $29$ $5$ $99$ $31$ $146$ $1$ $2$ $7$ $3$ $178$ $1$ $2$ $7$ $5$ $193$ $18$ $43$ $159$ $80$ $226$ $1$ $8$ $31$ $7$ $241$ $1$ $2$ $8$ $1$ $257$ $4$ $15$ $52$ $49$ $313$ $1$ $1$ $5$ $1$ $328$ $7$ $3$ $30$ $8$ $337$ $15$ $34$ $147$ $26$ $353$ $1$ $1$ $5$ $3$ $386$ $1$ $2$ $9$ $1$ $401$ $1$ $2$ $9$ $4$ $433$ $8$ $13$ $53$ $50$ $466$ $2$ $43$ $181$ $151$ $482$ $5$ $6$ $31$ $7$ $521$ $9$ $1$ $43$ $1$ $562$ $10$ $11$ $61$ $7$ $577$ $175$ $188$ $929$ $848$ $578$ $24$ $37$ $187$ $85$ $593$ $1$ $2$ $10$ $3$ $6673$ $161$ $26$ $820$ $139$ $706$ $17$ $28$ $149$ $13$ $712$ $15$ $13$ $8$	17	5	6	13	8
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	41	1	1	3	1
1131265136114213729599311461273178127519318431598022618317241128125741552493131151328733083371534147263531153386129140112944338135350466243181151482563175219143156210116175771751889298485782437187855931210362616291419764116199778673161268201397061728149137121513863676173371177611648021105319857753923	82	219	320	1011	247
1361142 $137$ $29$ $5$ $999$ $31$ $146$ 12 $7$ $3$ $178$ 1 $2$ $7$ $5$ $193$ $18$ $43$ $159$ $80$ $226$ 1 $8$ $31$ $7$ $241$ 1 $2$ $8$ $1$ $257$ $4$ $15$ $52$ $49$ $313$ 11 $5$ $1$ $328$ $7$ $3$ $30$ $8$ $337$ $15$ $34$ $147$ $26$ $353$ 11 $5$ $3$ $386$ 1 $2$ $9$ $1$ $401$ 1 $2$ $9$ $4$ $433$ $8$ $13$ $53$ $50$ $466$ $2$ $43$ $181$ $151$ $482$ $5$ $6$ $31$ $7$ $521$ $9$ $1$ $43$ $1$ $562$ $10$ $11$ $61$ $7$ $577$ $175$ $188$ $929$ $848$ $578$ $24$ $37$ $187$ $85$ $593$ $1$ $2$ $10$ $3$ $626$ $16$ $29$ $141$ $97$ $641$ $16$ $19$ $97$ $78$ $673$ $161$ $26$ $820$ $139$ $706$ $17$ $28$ $149$ $13$ $712$ $15$ $13$ $86$ $36$ $761$ $7$ $5$ $39$ $23$ <t< td=""><td>97</td><td>10</td><td>37</td><td>112</td><td>71</td></t<>	97	10	37	112	71
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	113	1	2	6	5
1461273 $178$ 1275 $193$ $18$ $43$ $159$ $80$ $226$ 18 $31$ 7 $241$ 1281 $257$ 4 $15$ $52$ $49$ $313$ 1151 $328$ 73 $30$ 8 $337$ $15$ $34$ $147$ $26$ $353$ 1153 $386$ 1291 $401$ 1294 $433$ 8 $13$ $53$ $50$ $466$ 2 $43$ $181$ $151$ $482$ 56 $31$ 7 $521$ 91 $43$ 1 $562$ $10$ $11$ $61$ 7 $577$ $175$ $188$ $929$ $848$ $578$ $24$ $37$ $187$ $85$ $593$ 12 $10$ $3$ $626$ $16$ $29$ $141$ $97$ $641$ $16$ $19$ $97$ $78$ $673$ $161$ $26$ $820$ $139$ $706$ $17$ $28$ $149$ $13$ $712$ $15$ $13$ $86$ $36$ $761$ $7$ $5$ $39$ $23$ $866$ $1$ $2$ $11$ $3$ $881$ $7$ $31$ $169$ $9$ $898$ $1$ $2$ $11$ $5$ <	136	1	1	4	2
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	137	29	5	99	31
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	146	1	2	7	3
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	178	1	2	7	5
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	193	18	43	159	80
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	226	1	8	31	7
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	241	1	2	8	1
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	257	4	15	52	49
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	313	1	1	5	1
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	328	7	3	30	8
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	337	15	34	147	26
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	353	1	1	5	3
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	386	1	2	9	1
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	401	1		9	4
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	433			53	50
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	466		43	181	151
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	977	3	10	56	11

 $w^4)$ Tal

# 7. Final remarks

There is an interesting relationship between  $8(19^4 + 21^4) = 14^4 + 40^4$  and  $2(7^4 + 20^4) = 19^4 + 21^4$ 

Multiply both sides of  $2(7^4 + 20^4) = 19^4 + 21^4$  by  $2^3$ , then we obtain  $14^4 + 40^4 = 8(19^4 + 21^4)$ .

Thus an equation  $n(x^4 + y^4) = z^4 + w^4$  can be transform  $(nx)^4 + (ny)^4 = n^3(z^4 + w^4)$ .

We could not find a solution of  $n(x^4+y^4) = z^4+w^4$  for n = 34, 40, 50, 65, 73, 89 whre n < 100. They are not the form  $(m^4+n^4)/2$ . I don't know if they don't have a solution to begin with, or if they have a large solution.

#### References

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